MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2018/2019

DEM5018 – ENGINEERING MATHEMATICS 1

(Diploma in Electronic Engineering)

4 MARCH 2019
9.00 AM – 11.00 AM
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of **2** pages (excluding cover page and appendices).
2. Attempt **ALL FOUR** questions.
3. Write your answers in the answer booklet provided.
4. Formulae are provided in the appendix section.

QUESTION 1

a. The matrices P and Q are given by $P=\begin{bmatrix}1 & 1 & 2\\ 0 & 4 & 1\\ 1 & 2 & 1\end{bmatrix}$ and $Q=\begin{bmatrix}1 & 1 & 1\\ 2 & 1 & 0\\ 0 & 1 & 1\end{bmatrix}$.

i. Find PQ. (6 marks)

ii. Hence, show that PQ is non-singular. (3 marks)

iii. Prove that $\det PQ=(\det P)(\det Q)$. (5 marks)

b. Sketch the graph of $2x-3y=6$. (6 marks)

c. Find the slope between (4, 6) and (2, − 1). Then, sketch the graph. (5 marks)

[TOTAL 25 MARKS]

QUESTION 2

a. Find the exact value of $\cos 75°+\sin 15°$. (10 marks)

b. Verify the identity $\dfrac{\sec\theta-\cos\theta}{\sec\theta+\cos\theta}=\dfrac{\sin^2\theta}{1+\cos^2\theta}$. (4 marks)

c. Given the complex number $z=5\angle 100°$,

i. convert to Cartesian form. (6 marks)

ii. plot the point in the complex plane. (2 marks)

d. If $z = 3 - 4i$ and $w = -2 + 2i$, write $\overline{z+w}$ expression in standard form. (3 marks)

[TOTAL 25 MARKS]

Continued ...

QUESTION 3

a. Given that $f(x) = 9x - 5$ and $g(x) = 2x + 3$.

i. Show that $f \circ g \neq g \circ f$. (5 marks)

ii. Find the function h if $g \circ h = x^2 + 3$. (4 marks)

iii. If $h \circ g = 3$, find the possible values of x. Give your answer correct to three decimal places. (6 marks)

b. The function f is defined as $k(x) = \dfrac{2}{x-3}$, $x \neq 3$. Find the inverse function k, $k^{-1}(x)$ in term of x. (5 marks)

c. The function P is defined by

$$P(x) = \begin{cases} \dfrac{x}{x-2} & , \quad x > 2 \\ x - 2 & , \quad x < 2 \end{cases}$$

Sketch the graph of function P. (5 marks)

[TOTAL 25 MARKS]

QUESTION 4

a. A curve is defined implicitly by the equation $x^2 + xy + y^2 = 3$.

i. Show that $\dfrac{dy}{dx} + \dfrac{2x+y}{x+2y} = 0$. (7 marks)

ii. Find the gradients of the curve at the points where the curve crosses the x-axis and y-axis. (4 marks)

iii. Show that the coordinates of the stationary points of the curve are $(-1,2)$ and $(1,-2)$. (7 marks)

b. The volume of water, $V\ cm^3$, in a container is given by $V = \dfrac{1}{3}h^3 + 9h$, where h is the height of the water in the container. Water is poured into the container at the rate of $8\ cm^3 s^{-1}$. Find the rate of change of the height of water, in $cm\ s^{-1}$, at the instant when its height is $3\ cm$. (7 marks)

[TOTAL 25 MARKS]

End of Page.

APPENDICES

Standard Form of Circle Equation, $(x-h)^2+(y-k)^2=r^2$

Fundamental Identities for Trigonometric Functions

***Even & Odd Properties*:**

$\sin(-\theta)=-\sin\theta$, $\csc(-\theta)=-\csc\theta$, $\cos(-\theta)=\cos\theta$,
$\sec(-\theta)=\sec\theta$, $\tan(-\theta)=-\tan\theta$, $\cot(-\theta)=-\cot\theta$

Reciprocal identities:

$$\csc\theta=\frac{1}{\sin\theta},\qquad \sec\theta=\frac{1}{\cos\theta},\qquad \cot\theta=\frac{1}{\tan\theta}$$

Pythagorean identities:

$$\sin^2\theta+\cos^2\theta=1,\qquad \tan^2\theta+1=\sec^2\theta,\qquad 1+\cot^2\theta=\csc^2\theta$$

Sum/Difference Formula for Cosine: $\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$

Sum/Difference Formula for Sine: $\sin(A\pm B)=\sin A\cos B\pm\sin B\cos A$

Sum/Difference Formula for Tangent: $\tan(A\pm B)=\dfrac{\tan A\pm\tan B}{1\mp\tan A\tan B}$

Double Angle Formula:

$$\cos 2A=\cos^2 A-\sin^2 A=1-2\sin^2 A=2\cos^2 A-1 \quad,\quad \sin 2A=2\sin A\cos A \quad,\quad \tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Half Angle Formula:

$$\sin\left(\frac{\alpha}{2}\right)=\pm\sqrt{\frac{1-\cos\alpha}{2}}\ ,\quad \cos\left(\frac{\alpha}{2}\right)=\pm\sqrt{\frac{1+\cos\alpha}{2}}\ ,\quad \tan\left(\frac{\alpha}{2}\right)=\pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$$

Complex Numbers

Relations between Polar and Rectangular Coordinates

$$x=r\cos\theta;\quad y=r\sin\theta;\quad r=\sqrt{x^2+y^2};\quad \tan\theta=\frac{y}{x}$$

Products and Quotients in Polar Form

$$z_1z_2=r_1r_2[\cos(\theta_1+\theta_2)+i\sin(\theta_1+\theta_2)]$$

$$\frac{z_1}{z_2}=\frac{r_1}{r_2}[\cos(\theta_1-\theta_2)+i\sin(\theta_1-\theta_2)];\quad z_2\neq 0$$

DeMoivre's Theorem

Powers of Complex Numbers on Polar Form: $z^n=r^n(\cos n\theta+i\sin n\theta)$

Roots of Complex Numbers in Polar Form:

$$z_k=\sqrt[n]{r}\left[\cos\left(\frac{\theta}{n}+\frac{2\pi k}{n}\right)+i\sin\left(\frac{\theta}{n}+\frac{2\pi k}{n}\right)\right]\quad \text{where}\quad k=0,1,2,\ldots,n-1$$

Differentiation Rules

Derivatives: $f'(a)=\lim_{h\to 0}\dfrac{f(a+h)-f(a)}{h}$

General Formulae

1. $\dfrac{d}{dx}[f(x)g(x)]=f(x)g'(x)+g(x)f'(x)$
2. $\dfrac{d}{dx}\left[\dfrac{f(x)}{g(x)}\right]=\dfrac{g(x)f'(x)-f(x)g'(x)}{[g(x)]^2}$
3. $\dfrac{d}{dx}(x^n)=nx^{n-1}$
4. $\dfrac{d}{dx}[g(x)]^n=n[g(x)]^{n-1}\cdot g'(x)$

Exponential and Logarithmic Functions

1. $\dfrac{d}{dx}(e^x)=e^x$
2. $\dfrac{d}{dx}(a^x)=a^x\ln a$
3. $\dfrac{d}{dx}(\ln x)=\dfrac{1}{x}$
4. $\dfrac{d}{dx}(\log_a x)=\dfrac{1}{x\ln a}$

Trigonometric Functions

1. $\dfrac{d}{dx}(\sin x)=\cos x$
2. $\dfrac{d}{dx}(\cos x)=-\sin x$
3. $\dfrac{d}{dx}(\tan x)=\sec^2 x$
4. $\dfrac{d}{dx}(\csc x)=-\csc x\cot x$
5. $\dfrac{d}{dx}(\sec x)=\sec x\tan x$
6. $\dfrac{d}{dx}(\cot x)=-\csc^2 x$

Inverse Trigonometry Function

1. $\dfrac{d}{dx}[\sin^{-1}x]=\dfrac{1}{\sqrt{1-x^2}}$
2. $\dfrac{d}{dx}[\cos^{-1}x]=-\dfrac{1}{\sqrt{1-x^2}}$
3. $\dfrac{d}{dx}[\tan^{-1}x]=\dfrac{1}{1+x^2}$
4. $\dfrac{d}{dx}[\csc^{-1}x]=-\dfrac{1}{x\sqrt{x^2-1}}$
5. $\dfrac{d}{dx}[\sec^{-1}x]=\dfrac{1}{x\sqrt{x^2-1}}$
6. $\dfrac{d}{dx}[\cot^{-1}x]=-\dfrac{1}{1+x^2}$

Hyperbolic Function

1. $\dfrac{d}{dx}[\sinh x]=\cosh x$
2. $\dfrac{d}{dx}[\cosh x]=\sinh x$
3. $\dfrac{d}{dx}[\tanh x]=\sec h^2 x$
4. $\dfrac{d}{dx}[\sec hx]=-\sec hx\tanh x$
5. $\dfrac{d}{dx}[\coth x]=-\csc h^2 x$
6. $\dfrac{d}{dx}[\csc hx]=-\csc hx\coth x$

The Second Derivative Test

Suppose f'' is continuous near c.

If $f'(c)=0$ and $f''(c)>0$, then f has local minimum at c.

If $f'(c)=0$ and $f''(c)<0$, then f has local maximum at c.